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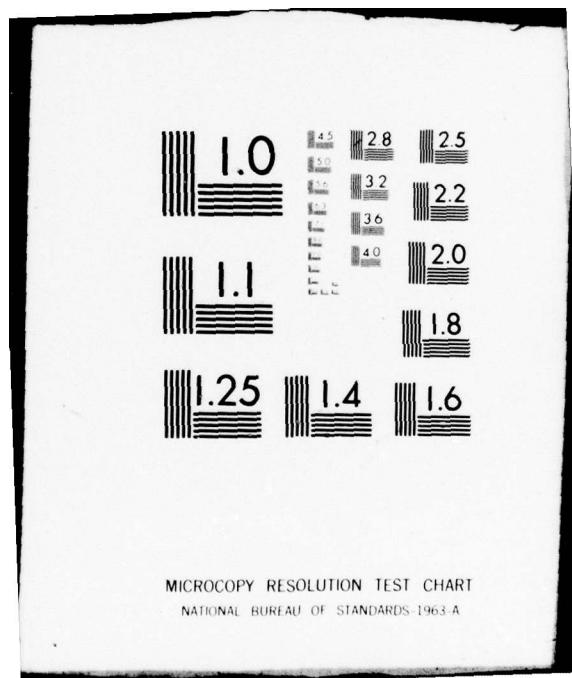
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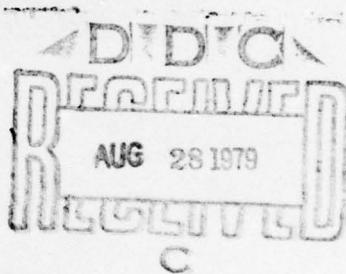
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BACKSCATTERING FROM AN ABSORBENT SPHERE

G. C. Lauchle

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The Pennsylvania State University  
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**Subject:** Backscattering From An Absorbent Sphere

**References:** See page 8.

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**Acknowledgement:** The curves presented in this letter were generated at the suggestion of Prof. E. J. Skudrzyk. Support has been through the Naval Sea Systems Command, Code 03.

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## INTRODUCTION

Backscattering cross section curves for hard and soft spheres may be found in many references, e.g., Bowman, et. al. [1]. Analogous curves for the absorbent sphere appear to be limited except for special cases of the surface impedance [2, 3]. In this letter we consider the simple case of a sphere whose surface is one of local reaction. That is, the motion of the surface at a point, due to some pressure at that point, is not coupled with the motions of other points on the surface. Many acoustical absorbents satisfy this definition. The surface is characterized in terms of the specific admittance,  $\beta$ , assumed independent of frequency. This admittance is related to the point impedance,  $z$ , by  $\beta = pc/z$ , where  $pc$  is the characteristic impedance of the acoustic medium. Numerical results for the backscattering cross section are presented herein as a function of acoustic radius,  $ka$ , and for discrete values of  $\beta$ .

## ANALYSIS

Sound scattering analyses for absorbent spheres may be found in the books by Morse and Ingard [4] and Skudrzyk [5], but neither author presents numerical examples of their analysis. We repeat the analysis here only for the sake of completeness.

The incident pressure due to a plane wave is given in the usual spherical coordinate notation by:

$$p_i = \bar{A} \sum_{n=0}^{\infty} (-i)^n (2n+1) P_n(\cos\theta) j_n(kr) , \quad (1)$$

where  $P_n$  is the Legendre polynomial and  $j_n$  the spherical Bessel function.

For  $\exp(-i\omega t)$  time dependence, the scattered pressure field is of the form:

$$p_s = \sum_{n=0}^{\infty} A_n P_n(\cos\theta) h_n^{(1)}(kr) , \quad (2)$$

where  $h_n^{(1)}$  is the spherical Hankel function of the first kind, and  $A_n$  is determined from the boundary condition at  $r=a$ ; namely,

$$\frac{\partial p}{\partial r} + ik\beta p \Big|_{r=a} = 0 . \quad (3)$$

Solving for  $A_n$  and making the far-field approximation:

$$h_n^{(1)}(kr) \xrightarrow[kr \rightarrow \infty]{} i^{-n-1} \exp(ikr)/kr , \quad (4)$$

we find that

$$p_s \xrightarrow{} \bar{A} \exp(ikr) \Phi(\theta)/r , \quad (5)$$

where  $\Phi(\theta)$  is the angle distribution function given by:

$$\Phi(\theta) = \frac{i}{k} \sum_{n=0}^{\infty} (-1)^n (2n+1) P_n(\cos\theta) \left\{ \frac{i\beta j_n(ka) + j'_n(ka)}{i\beta h_n^{(1)}(ka) + h_n^{(1)}(ka)} \right\} . \quad (6)$$

The primes on the spherical functions denote differentiation with respect to the argument.

The backscattering cross section is a measure of the sound intensity scattered back in the direction of the incident plane wave. When normalized to the intensity of the incident wave, we have

$$\sigma_o = 2\pi |\Phi(0)|^2 \int_0^{\pi} \sin\theta d\theta = 4\pi |\Phi(0)|^2 . \quad (7)$$

The relative backscattering cross section is eq. (7) normalized by the area of the geometric shadow,  $\pi a^2$ ; hence,

$$\frac{\sigma_o}{\pi a^2} = \frac{4}{(ka)^2} \left| \sum_{n=0}^{\infty} (-1)^n (2n+1) \left\{ \frac{j_n'(ka) + i\beta j_n(ka)}{h_n^{(1)}(ka) + i\beta h_n^{(1)}(ka)} \right\} \right|^2 . \quad (8)$$

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### NUMERICAL RESULTS

Equation (8) has been evaluated for  $0.2 \leq ka \leq 21$  and for a few discrete values of  $\beta$  including  $\beta = 0$  ( $z \rightarrow \infty$ ) which is the rigid case,  $\beta \rightarrow \infty$  ( $z = 0$ ) which is the pressure release case, and  $\beta = 1$  ( $z = pc$ ) which is the  $pc$ -match case. The spherical Bessel and Neumann functions were calculated on an IBM 370 digital computer using the method of Corbato' and Uretsky [6]. The series of equation (8) was found to converge in about  $[ka]$  terms when  $ka > \pi$ , although six terms were used when  $ka \leq 6$ , and  $[ka]$  terms when  $ka > 6$ .

Figure 1 shows the results of these computations for  $0.2 \leq ka \leq 21$ , while Figure 2 shows only the low-frequency behavior ( $0.2 \leq ka \leq 2.4$ ). We see from Figure 1 that for a large sphere (large  $ka$ ), the backscattering decreases as the specific admittance is increased from zero to one. When  $\beta$  is increased to values greater than one, the backscattering cross section is observed to increase. Figure 3 demonstrates this behavior for  $ka = 21$ . We note that for  $ka > 2$ , the rigid and pressure release spheres reflect approximately equal energy.

At low values of  $ka$  (small spheres), the behavior is nearly reversed of what occurs for large spheres. For  $ka \leq 0.6$ , the rigid sphere is the worst reflector while the pressure release one is the best. In between, the magnitude of the backscattering cross section increases monotonically with  $\beta$  (Figure 3); there is no null corresponding to the case when the characteristic impedance of the absorbent matches that of the acoustic medium.

### CONCLUSIONS

It has been the purpose of this letter to present a series of curves for the backscattering cross section of absorbent spheres. In the two limiting cases of the ideally rigid and pressure release spheres, this

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cross section asymptotically approaches the area of the geometric shadow when the wavelength becomes small. When the point impedance of the surface matches the characteristic impedance of the medium, the backscattering cross section decreases with  $ka$  by approximately a factor of four for each octave in  $ka$  (we should not assume this to be generally true for  $ka > 21$ ). In general, for  $ka \geq 5$ , the backscattering cross section increases with increasing  $|\beta - 1|$ , where  $0 \leq \beta \leq \infty$ . For small spheres ( $ka \leq 0.6$ ),  $\sigma_0$  increases monotonically with  $\beta$ . For intermediate values of  $ka$ , corresponding to sphere radii on the order of a wavelength, the backscattering cross section oscillates with  $ka$  and no generalization regarding its dependence on  $\beta$  may be made.

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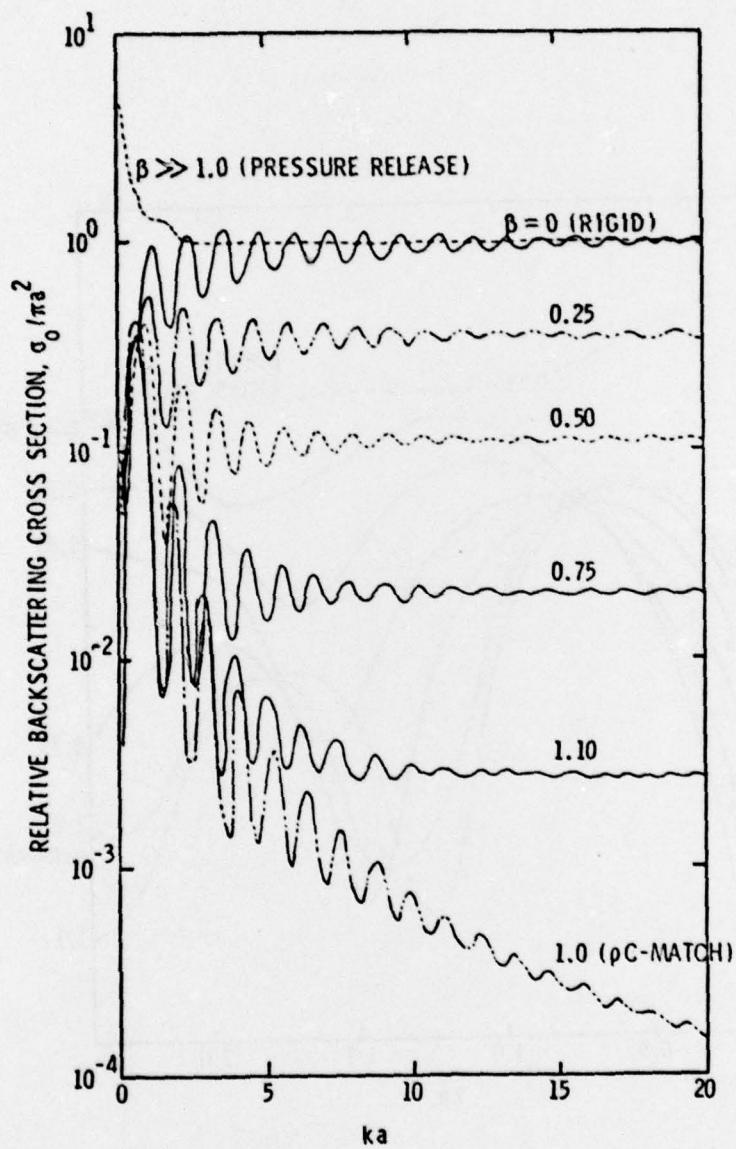


Figure 1 - Backscattering cross section curves for various values of  $\beta$  and for  $0.2 \leq ka \leq 21$ .

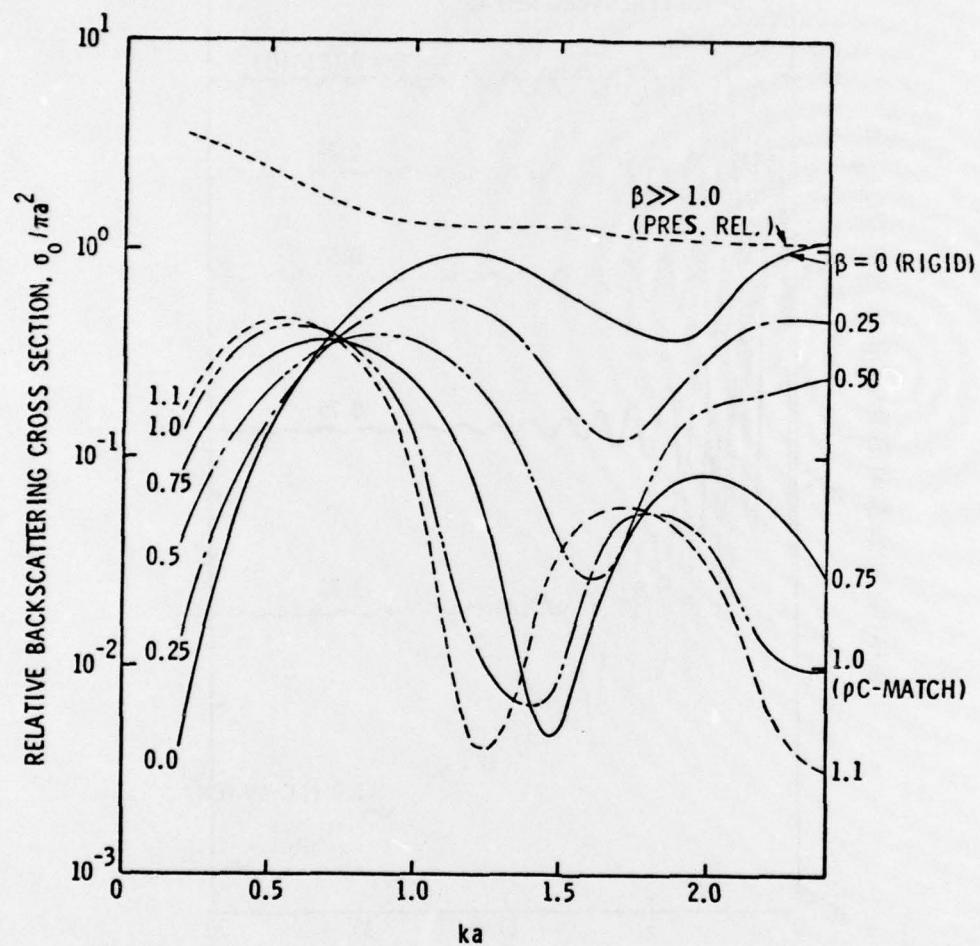


Figure 2 - Backscattering cross section curves for various values of  $\beta$  and for  $0.2 \leq ka \leq 2.4$ .

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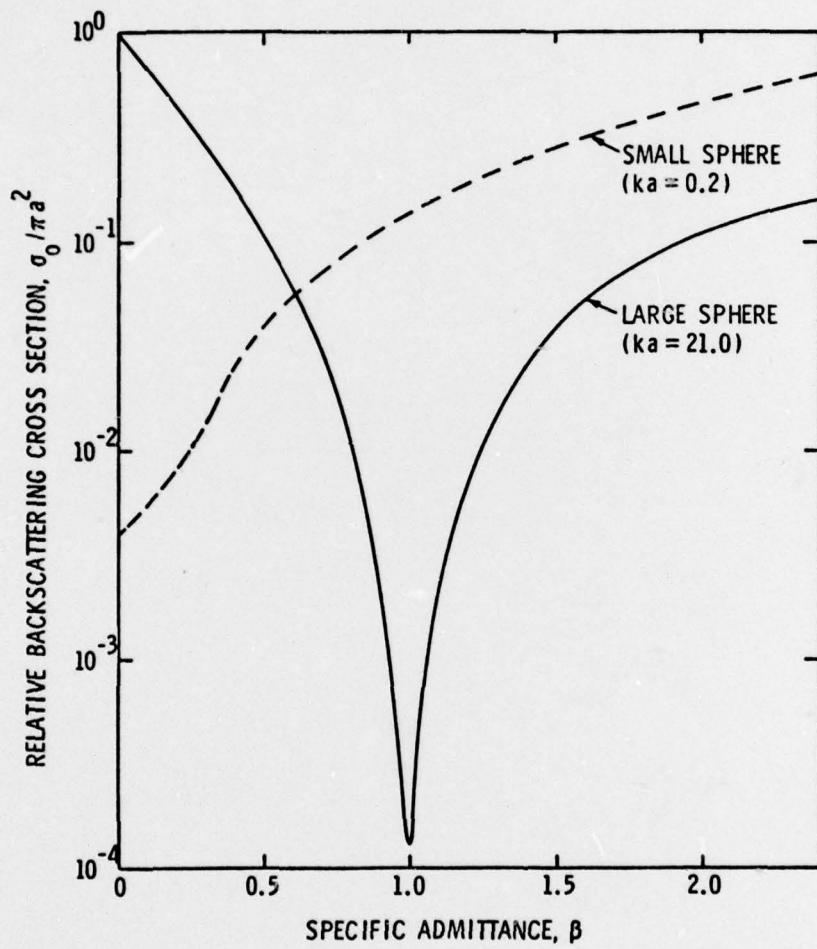


Figure 3 - The variation of the backscattering cross section with  $\beta$  for both large and small spheres.

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